

KEK-TH-684  
 hep-ph/0004025

**$\mu \rightarrow e\gamma$  AND  $\mu \rightarrow 3e$  PROCESSES WITH POLARIZED MUON  
 AND SUPERSYMMETRIC GRAND UNIFIED THEORIES <sup>a</sup>**

Ken-ichi Okumura

*Theory Group, KEK, Tsukuba, Ibaraki, 305-0801 Japan  
 and*

*Department of Accelerator Science, The Graduate University for Advanced Studies,  
 Tsukuba, Ibaraki, 305-0801 Japan,  
 E-mail: ken-ichi.okumura@kek.jp*

Lepton flavor violating processes  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow 3e$  with polarized muons are studied in the supersymmetric grand unified theory (SUSY GUT). As a result of a detailed numerical calculation, it is shown that the P- and T-odd asymmetries defined with the help of the muon polarization and the ratio of two branching fractions make a good contrast between the SU(5) and SO(10) SUSY GUT. These observables are useful to extract differences of the two theories. In particular, the P-odd asymmetry of  $\mu \rightarrow e\gamma$  varies 100%–100% in SO(10) whereas it is 100% in SU(5) and the T-odd asymmetry of  $\mu \rightarrow 3e$  can reach 15% in SU(5) within the EDM constraints whereas it is small in SO(10).

## 1 Introduction

In the standard model (SM) lepton flavor is conserved because the matter contents of the SM and the gauge symmetry forbid lepton flavor violating renormalizable couplings. However, matter contents beyond the SM can easily accommodate violation of lepton flavor. In the supersymmetric (SUSY) extension of the SM, scalar partners of the ordinary leptons, which are introduced to solve the gauge hierarchy problem, become a source of lepton flavor violation (LFV). Terms in the SUSY Lagrangian which violate SUSY without the quadratic divergence (the soft SUSY breaking terms) are not necessarily diagonal with respect to the flavor indices and become a source of LFV. Even if their origin is flavor-blind at a very high energy scale as in the case of the minimal supergravity model (minimal SUGRA), radiative corrections from LFV interactions between that scale and the electro-weak (EW) scale can induce LFV off-diagonal elements in the slepton mass matrices <sup>2</sup>. At a low energy

---

<sup>a</sup>Talk given at the Workshop on High Intensity Muon Sources (HIMUS99), Tsukuba, Japan, 1-4 Dec 1999.

scale, an existence of such interactions can appear as LFV processes such as  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow 3e$  through loop diagrams including SUSY partners suppressed only by the power of their mass scale. Then, the precise measurement of such a low energy process is a sensitive probe for the LFV interactions at a very high energy scale. In particular, grand unified theory (GUT) predicts unification of quarks and leptons and the flavor mixing in quark sector means that GUT interactions violate lepton flavor conservation. The pattern of the LFV interactions reflects the structure of SUSY GUT. LFV processes which are induced from such interactions can be just below the current experimental bounds<sup>3,4</sup>. There are experimental projects which aim to explore the region of more than three orders of magnitude below the current limits<sup>5</sup>. In this article we discuss the possibility to use various P- and T- odd asymmetries in  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow 3e$  with polarized muons to study the LFV interactions above the GUT scale and distinguish different SUSY models: SU(5) and SO(10) SUSY GUT<sup>b</sup>.

## 2 $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ processes in SU(5) and SO(10) SUSY GUT

First, we introduce the minimal SU(5) SUSY GUT<sup>6</sup> and discuss the qualitative feature of LFV in this theory. In the SU(5) SUSY GUT, all the matter fields in the minimal supersymmetric standard model (MSSM) are embedded in three generations of **10** representation  $T_i(Q_i, U_i^c, E_i^c)$  and  **$\bar{5}$**  representation  $\bar{F}_i(D_i^c, L_i)$  of SU(5). Two Higgs doublets are embedded in **5** ( $H(H_C, H_2)$ ) and  **$\bar{5}$**  ( $\bar{H}(\bar{H}_C, H_1)$ ) representations with newly introduced colored Higgs fields  $H_C$  and  $\bar{H}_C$ . The renormalizable Yukawa superpotential is written as follows.

$$\mathcal{W}_{SU(5)} = \frac{1}{8} \epsilon_{abcde} (\hat{y}_u)_i T_i^{ab} T_i^{cd} H^e + (y_d)_{ij} \bar{F}_{ia} T_j^{ab} \bar{H}_b, \quad (1)$$

where we choose the basis in which the up-type Yukawa coupling constant  $y_u$  is diagonal. The important point in this formula for LFV is that the right-handed leptons have GUT interaction through up-type Yukawa coupling constant:  $(\hat{y}_u)_i U_i^c E_i^c H_C$ . Because of this GUT interaction including the large top Yukawa coupling constant, even if the soft SUSY breaking terms have the minimal-SUGRA-type universal structure at the Planck scale, radiative corrections between the Planck scale and the GUT scale reduces the third generation slepton mass compared to the first and the second generation. The mass difference can be approximated as follows:

$$\Delta m^2 \simeq \frac{3}{8\pi^2} |(\hat{y}_u)_3|^2 m_0^2 (3 + |A_0|^2) \ln\left(\frac{M_P}{M_G}\right). \quad (2)$$

---

<sup>b</sup>This talk is based on the work in reference<sup>1</sup>.

In order to discuss the LFV processes, it is convenient to go to the basis where the lepton Yukawa coupling constant is diagonal.

$$V_R y_e V_L^\dagger = \text{diagonal} \quad (3)$$

where  $V_R$  and  $V_L$  are unitary matrices. In this basis, the right-handed sleptons have LFV off-diagonal elements in the mass matrix as follows:

$$(m_E^2)_{ij} \simeq -(V_R^T)_{3i}(V_R^\dagger)_{3j}\Delta m^2. \quad (4)$$

A similar formula can be obtained for the left-right mixing mass matrix which is induced from the trilinear scalar coupling in the soft SUSY breaking terms. All the amplitudes of  $\mu^+ \rightarrow e^+\gamma$  and  $\mu^+ \rightarrow e^+e^+e^-$  processes pick up these off diagonal elements and they are proportional to the combination of unitary matrix elements  $\lambda_\tau = (V_R^T)_{32}(V_R^\dagger)_{31}$ . In the minimal model,  $V_R$  is written by the transposed of the CKM matrix because of the GUT relation  $y_e = y_d^T$ . However, this relation can not explain the ratio of the first and second generation quark and lepton masses. In the realistic model, higher dimensional operators at the GUT scale can explain such a mismatch. With these new operators,  $V_R$  can be different from the corresponding CKM matrix element and the branching ratios themselves have considerable model dependence. Here, instead of dealing with a detail of models we focus on the observables which are expressed by the ratio of amplitudes and insensitive to the value of  $\lambda_\tau$ .

Next we explain the case of the minimal SO(10) SUSY GUT<sup>7</sup>. In this theory, all the matter fields in the MSSM are unified in three generations of 16 representation of SO(10) ( $\Psi_i$ ). We introduce two 10 representation Higgs fields  $\Phi_u$ ,  $\Phi_d$  for the up- and down-type Yukawa couplings to reproduce the CKM matrix. The renormalizable Yukawa superpotential can be written as follows<sup>8</sup>:

$$\mathcal{W}_{SO(10)} = \frac{1}{2}(\hat{y}_u)_i \Psi_i \Phi_u \Psi_i + \frac{1}{2}(y_d)_{ij} \Psi_i \Phi_d \Psi_j. \quad (5)$$

The important difference from the SU(5) case is that the left-handed leptons also have GUT interaction through the up-type Yukawa coupling constant. Because of this GUT interaction the left-handed sleptons also have LFV off-diagonal elements in the mass matrix  $(m_L)_{ij}$  even if we have the universal scalar mass at the Planck scale.

The LFV off-diagonal elements of the slepton mass matrix discussed above induce  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow 3e$  through loop diagrams which include sleptons, neutralino and chargino. The most general form of the effective Lagrangian which describes these processes can be parameterized using the Lorentz invariance,

the gauge invariance and the Fierz rearrangement as follows:

$$\begin{aligned}
\mathcal{L} = & -\frac{4G_F}{\sqrt{2}} \{ m_\mu A_R \overline{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L \overline{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} \\
& + g_1 (\overline{\mu}_R e_L) (\overline{e}_R e_L) + g_2 (\overline{\mu}_L e_R) (\overline{e}_L e_R) \\
& + g_3 (\overline{\mu}_R \gamma^\mu e_R) (\overline{e}_R \gamma_\mu e_R) + g_4 (\overline{\mu}_L \gamma^\mu e_L) (\overline{e}_L \gamma_\mu e_L) \\
& + g_5 (\overline{\mu}_R \gamma^\mu e_R) (\overline{e}_L \gamma_\mu e_L) + g_6 (\overline{\mu}_L \gamma^\mu e_L) (\overline{e}_R \gamma_\mu e_R) + h.c. \}, \quad (6)
\end{aligned}$$

where  $A_R$  and  $A_L$  are the photon-penguin type coupling constants which contribute to both the  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow 3e$  processes and  $g_{1-6}$  are the four-fermion type coupling constants which contribute to only the  $\mu \rightarrow 3e$  process. The Fermi constant  $G_F$  and the muon mass  $m_\mu$  is factored out and these coupling constants are dimensionless. These effective coupling constants have a specific pattern for the two SUSY GUTs. In the SU(5) case, only the right-handed sleptons have LFV coupling constants. Then,  $A_R$  is suppressed by  $m_e/m_\mu$  relative to  $A_L$  because a chirality flip must occur at the electron side. Among the four-fermion type coupling constants,  $g_1$  and  $g_2$  are suppressed relative to the other coupling constants by the Yukawa coupling constant because they need a chirality flip. Then,  $g_3$  and  $g_5$  dominate the process. On the other hand, in the SO(10) case, both the left-handed and right-handed sleptons have LFV coupling constants. As a consequence, photon-penguin type diagrams which pick up the tau mass as a chirality flip become possible because the slepton in the loop diagram can change its flavor before and after the chirality flip<sup>4</sup>. They are enhanced by  $m_\tau/m_\mu$  compared to the other diagrams. Then,  $A_R$  and  $A_L$  dominate the  $\mu \rightarrow 3e$  process.

### 3 P- and T-odd asymmetries in $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ processes

Now that we explained the qualitative features of LFV effective coupling constants in the SU(5) and SO(10) SUSY GUT, let us define  $\lambda_\tau$  insensitive observables: the P- and T-odd asymmetries. In the case of the  $\mu^+ \rightarrow e^+ \gamma$  process, the differential branching ratio is written as follows:

$$\frac{dB(\mu \rightarrow e\gamma)}{d\cos\theta} = \frac{B(\mu \rightarrow e\gamma)}{2} \{1 + A(\mu \rightarrow e\gamma) P \cos\theta\}, \quad (7)$$

$$A(\mu \rightarrow e\gamma) = \frac{|A_L|^2 - |A_R|^2}{|A_L|^2 + |A_R|^2}, \quad (8)$$

where  $\theta$  is an angle between the decay positron momentum and the initial muon polarization. We define the coefficient of the angle dependence as the P-odd asymmetry  $A(\mu \rightarrow e\gamma)$ . In the case of the  $\mu^+ \rightarrow e^+ e^+ e^-$  process, the

kinematics is determined by energies of two decay positrons  $E_1, E_2$  ( $E_1 > E_2$ ) and two angle which indicate the direction of muon polarization to the decay plane  $\Omega(\theta, \phi)$  (See Figure 1 (a)). In the coordinate of Figure 1 (a), components of the initial muon polarization are written using the momenta of the decay positron and electron as follows:

$$\vec{P} = \left( \vec{P} \cdot \frac{\vec{p}_1 - (\vec{p}_1 \cdot \frac{\vec{p}_3}{|\vec{p}_3|}) \frac{\vec{p}_3}{|\vec{p}_3|}}{|\vec{p}_1 - (\vec{p}_1 \cdot \frac{\vec{p}_3}{|\vec{p}_3|}) \frac{\vec{p}_3}{|\vec{p}_3|}|}, \vec{P} \cdot \frac{\vec{p}_3 \times \vec{p}_1}{|\vec{p}_3 \times \vec{p}_1|}, \vec{P} \cdot \frac{\vec{p}_3}{|\vec{p}_3|} \right). \quad (9)$$

The x and z components of the polarization are P-odd because the momenta change their sign under the parity (P) operation and the polarization does not. The y component of the polarization is T-odd because the momenta and the polarization change their sign under the time reversal (T) operation. Then three asymmetries of branching ratio can be defined relative to the muon polarization as follows:

$$A_i = \frac{\int_{P_i > 0} d\Omega \frac{dB(\mu \rightarrow 3e)}{d\Omega} - \int_{P_i < 0} d\Omega \frac{dB(\mu \rightarrow 3e)}{d\Omega}}{B(\mu \rightarrow 3e)}, \quad (i = x, y, z). \quad (10)$$

We call the asymmetry relative to the z component as  $A_{P_1}$ , that of the x component as  $A_{P_2}$  and that of the y component as  $A_T$ . These asymmetries can be written in terms of combinations of the effective coupling constants. The P-odd asymmetries  $A_{P_1}$  and  $A_{P_2}$  reflect a chiral structure of these coupling constants. The T-odd asymmetries can be induced only through the interference between the photon-penguin and four-fermion type coupling constants as follows:

$$A_T \equiv A_y \simeq \frac{3[2\text{Im}(eA_R g_4^* + eA_L g_3^*) - 1.6\text{Im}(eA_R g_6^* + eA_L g_5^*)]}{2B(\mu \rightarrow 3e)}, \quad (11)$$

where we set a cut-off in the positron energy ( $E_{1,2} < 0.04m_\mu$ ) because near the kinematical edge ( $E_{1,2} \simeq \frac{m_\mu}{2}$ ) the contribution to the denominator from the photon-penguin amplitudes has a logarithmic singularity and dilutes  $A_T$ .

#### 4 Results of numerical calculation

Now, let us show the results of our numerical calculations for these asymmetries. In the actual numerical analysis, we solved the renormalization group equations from the Planck scale to the EW scale with full flavor mixings. We assumed the minimal-SUGRA-type universal boundary conditions for the soft SUSY breaking parameters at the Planck scale. All the scalar fields have a mass

$m_0$  and the gaugino has a mass  $M_0$ . The trilinear scalar coupling constants are proportional to the Yukawa coupling constants with a universal coefficient  $m_0 A_0$ . In general, these parameters can have complex phases and two of them are physically independent. We take the phase of  $A_0$  ( $\theta_{A_0}$ ) and the phase of the  $\mu$  term ( $\theta_\mu$ ) in the MSSM superpotential as SUSY CP violating phases. We also assumed the radiative EW symmetry breaking. Then, the free parameters of the two SUSY GUTs are  $m_0$ ,  $M_0$ ,  $|A_0|$ ,  $\theta_{A_0}$ ,  $\theta_\mu$  and the ratio of two VEVs of the Higgs doublets  $\tan\beta$ . The SUSY CP violating phases induce the electron, neutron and atomic EDMs through one loop diagrams. We included such EDM constraints. Phenomenological constraints from LEP, Tevatron and  $b \rightarrow s\gamma$  decay are also imposed.

First we show the results of the SU(5) SUSY GUT with no SUSY CP violating phase in Figure 1 (b)-(d). In this model,  $A_L$ ,  $g_3$  and  $g_5$  give sizable contributions. SUSY parameters are fixed as  $\tan\beta = 3$  and  $M_2 = 150$  GeV. The ratio of the two branching fractions  $B(\mu \rightarrow 3e)/B(\mu \rightarrow e\gamma)$  is known to become constant ( $\simeq 0.0061$ ) when the photon-penguin diagram dominates the  $\mu \rightarrow 3e$  process. In our numerical calculation, it is enhanced in a wide region of parameter space compared to the photon-penguin dominant case because of the contribution of the four-fermion type coupling constants. In a large parameter space, the ratio is enhanced by more than five. Figure 1 (b) show the branching ratio for  $\mu \rightarrow 3e$  normalized by  $|\lambda_\tau|^2$ . If we take  $V_R$  as corresponding CKM matrix elements,  $|\lambda_\tau|^2$  is approximately  $10^{-7}$ , however, if  $\lambda_\tau$  is enhanced by a factor of 10 due to higher dimensional operators for the Yukawa coupling constant at the GUT scale, the branching ratio becomes  $10^2$  times larger. We checked the P-odd asymmetry of  $\mu \rightarrow e\gamma$  becomes almost 100% as expected. Figure 1 (c), (d) show the P-odd asymmetries  $A_{P_1}$  and  $A_{P_2}$  of the  $\mu \rightarrow 3e$  process. The  $A_{P_1}$  varies from  $-30\%$  to  $40\%$  and  $A_{P_2}$  varies from  $-10\%$  to  $15\%$  parameter sensitively. It is interesting that the effective coupling constants  $g_3$ ,  $g_5$  and  $A_L$  can be determined up to the overall phase from  $B(\mu \rightarrow e\gamma)$ ,  $B(\mu \rightarrow 3e)$  and  $A_{P_1}$ . Then,  $A_{P_2}$  is predicted and can be used for checking the assumption. Next, we introduce the SUSY CP violating phases and show the contour plot of the T-odd asymmetry in Figure 1 (f). The SUSY parameters are chosen as  $\tan\beta = 3$ ,  $M_2 = 300$  GeV,  $\theta_{A_0} = \pi/2$ .  $\theta_\mu$  is constrained very severely by the EDM experiments and we consider a small allowed region around  $\theta_\mu = 0$ . Figure 1 (e) also shows the branching ratio of  $\mu \rightarrow 3e$  divided by  $|\lambda_\tau|$  for corresponding SUSY parameters. The black shaded region indicates excluded region by the current experimental bounds for the electron, neutron and Hg EDMs. Within the EDM constraints, it is shown that the T-odd asymmetry can become 15%.

Next, we show the results of the SO(10) SUSY GUT. In this case, the

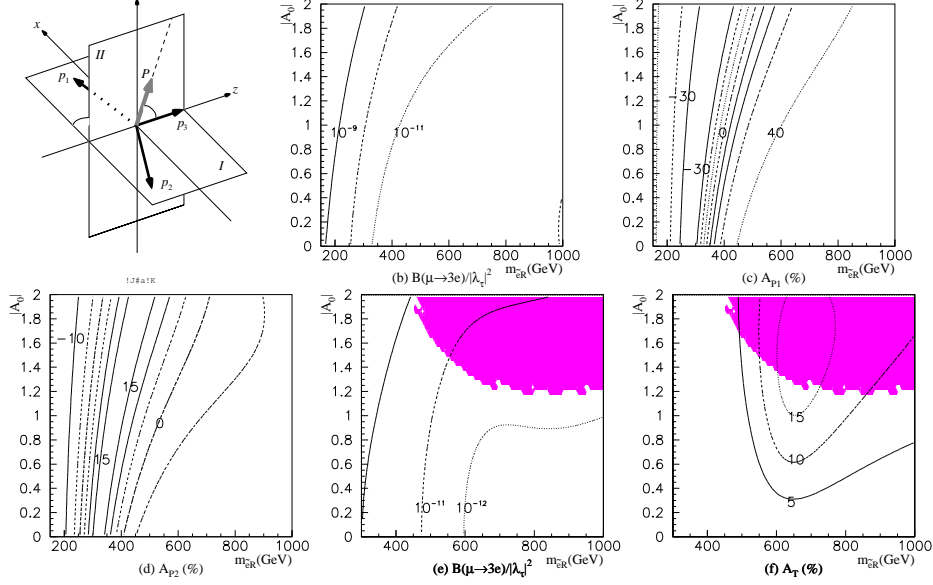


Figure 1: Branching ratio and P- and T-odd asymmetries of  $\mu^+ \rightarrow e^+e^+e^-$  in the SU(5) SUSY GUT for  $\tan\beta = 3$ . The SUSY parameters are fixed as  $M_0 = 150$  GeV and  $\theta_{A_0} = \theta_\mu = 0$  for (b)-(c) and  $M_0 = 300$  GeV,  $\theta_{A_0} = \frac{\pi}{2}$  and  $\theta_\mu = 0$  for (e), (f). The black shaded region is excluded by the experimental bounds for the neutron, electron and Hg EDMs.

photon-penguin type coupling constants  $A_L$  and  $A_R$  dominate the  $\mu \rightarrow 3e$  process. Figure 2 (a) shows  $B(\mu \rightarrow e\gamma)$  divided by  $|\lambda_\tau|^2$ . The SUSY parameters are fixed as  $\tan\beta = 3$ ,  $M_2 = 150$  GeV,  $\theta_{A_0} = 0$  and  $\theta_\mu = 0$ . We confirmed the ratio of two branching fraction is almost constant as expected. Figure 2 (b) shows the P-odd asymmetry of  $\mu \rightarrow e\gamma$ . The asymmetry varies from  $-20\%$  to  $-90\%$  and the absolute value becomes larger with the mass of right-handed selectron. This result can not be explained if only the diagrams enhanced by  $m_\tau$  dominate the process as believed previously. In such a case  $A_L$  and  $A_R$  have the same contribution. Instead we found that the chargino contribution which only contributes to  $A_R$  can not be neglected in spite of no  $m_\tau$  enhancement (See Figure 2 (c)). The chargino contribution even dominates the neutralino contribution enhanced by  $m_\tau$  when the scalar mass becomes large. This is mainly because the dominant diagram of the former contribution picks up a chirality flip with a factor  $m_\mu/m_W$  at the Higgsino vertex whereas the latter picks up it by the left-right mixing mass in the slepton internal line with a factor  $m_\tau/m_{\tilde{\tau}}$  which decreases with the scalar mass. The P-odd asymmetries

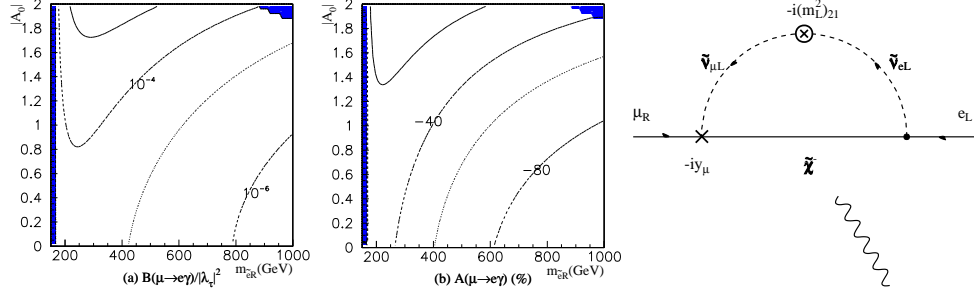


Figure 2: Branching ratio and P-odd asymmetry of  $\mu^+ \rightarrow e^+ \gamma$  in the SO(10) SUSY GUT for  $\tan \beta = 3$ . The SUSY parameters are fixed as  $M_0 = 150$  GeV and  $\theta_{A_0} = \theta_\mu = 0$ .

Table 1: Summary of the numerical calculation

	SU(5) SUSY GUT	SO(10) SUSY GUT
$\frac{B(\mu \rightarrow 3e)}{B(\mu \rightarrow e\gamma)}$	$0.007 - O(1)$	constant ( $\sim 0.0062$ )
$A(\mu \rightarrow e\gamma)$	$+100\%$	$+100\% - -100\%$
$A_{P_1}$	$-30\% - +40\%$	$A_{P_1} \simeq -\frac{1}{10} A(\mu \rightarrow e\gamma)$
$A_{P_2}$	$-20\% - +20\%$	$A_{P_2} \simeq -\frac{1}{6} A(\mu \rightarrow e\gamma)$
$ A_T $	$\lesssim 15\%$	$\lesssim 0.01\%$

$A_{P_1}$  and  $A_{P_2}$  are simply proportional to  $A(\mu \rightarrow e\gamma)$  because in the SO(10) GUT only the photon-penguin type coupling constants dominate the process and there is essentially only one P-odd asymmetry. The T-odd asymmetry is found small because the T-odd asymmetry occurs only through the interference terms of the photon-penguin and four-fermion type coupling constants and it is suppressed when the photon-penguin contributions dominate the process. Table 1 is the summary of our numerical calculation. These results make a good contrast between the two SUSY GUTs.

In this article we discussed the possibility to use various P- and T- odd asymmetries relative to the muon polarization in the  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow 3e$  to explore LFV interactions at the GUT scale. As a result of a detailed numerical calculation, we showed these observables and the ratio of two branching fractions are useful to extract the difference of the SU(5) and SO(10) SUSY GUT.

## References



1. Y. Okada, K. Okumura and Y. Shimizu, *Phys. Rev. D* **58**, 051901 (1998); *Phys. Rev. D* **61**, 094001 (2000)
2. L.J. Hall, V.A. Kostelecky and S. Raby, *Nucl. Phys. B* **267**, 415 (1986).
3. R. Barbieri and L.J. Hall, *Phys. Lett. B* **338**, 212 (1994)
4. R. Barbieri, L.J. Hall and A. Strumia, *Nucl. Phys. B* **445**, 219 (1995).
5. L.M. Barkov *et.al.*, *Research Proposal to PSI* (1999); M. Bachman *et.al.*, *Proposal to BNL AGS* (1997).
6. H. Georgi and S.L. Glashow, *Phys. Rev. Lett.* **32**, 438 (1974); S. Dimopoulos and H. Georgi, *Nucl. Phys. B* **193**, 150 (1981).
7. H. Fritzsch and P. Minkowski, *Ann.of Phys.***93**, 193 (1975); H. Georgi in *Particles and Fields*, ed. C.E. Carlson (APS/DPF Williamsburg, 1975).
8. S. Dimopoulos and L.J. Hall, *Phys. Lett. B* **344**, 185 (1995).